Magnetic Fields from Phase Transitions

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The generation of primordial magnetic fields from cosmological phase transitions is discussed, paying particular attention to the electroweak transition and to the various definitions of the 'average' field that have been put forward. It is emphasised that only the volume average has dynamical significance as a seed for galactic dynamos. On rather general grounds of causality and energy conservation, it is shown that, in the absence of MHD effects that transfer power in the magnetic field from small to large scales, processes occurring at the electroweak transition cannot generate fields stronger than 10^{-20} Gauss on a scale of 0.5 Mpc. However, it is implausible that this upper bound could ever be reached, as it would require all the energy in the Universe to be turned into a magnetic field coherent at the horizon scale. Non-linear MHD effects seem therefore to be necessary if the electroweak transition is to create a primordial seed field.

I. INTRODUCTION

The origin of galactic magnetic fields appears to lie in a primordial seed field, produced in the very early Universe. The primordial magnetic flux would have been frozen into the highly conductive plasma on scales sufficiently large that the diffusion of the magnetic field can be neglected, keeping constant the magnetic flux through a closed curve fixed in the plasma. This field would have been trapped and compressed in the collapsing protogalaxy, and then amplified to the observed intragalactic value, of order 10^{-6} G, by a dynamo effect during galaxy formation [1]. It is believed that the galactic dynamo needs only a very small field, perhaps of order 10^{-18} G to 10^{-21} G, to operate. Given that a galaxy represents an overdensity of order 10^{-6}, the primordial field as it would be measured today could be a factor 10^4 smaller.

There seems to be no shortage of ways in which fundamental processes in the early Universe could have produced such a seed field. Most fall into two classes, according to whether the field is produced during an inflationary era or during a phase transition, although a mechanism involving asymmetric neutrino emission from rotating black holes manages to avoid this classification [2].

Turner and Widrow [3] estimated the production of large-scale magnetic fields in a set of inflationary mod-els. Ordinary electromagnetism is conformally invariant, and therefore the initial vacuum state remains the vacuum during inflation, and there can be no particle production. They therefore considered a number of models with broken conformal invariance, but the only one which produced a large enough seed field did not respect gauge invariance. Ratra [4] considered what is perhaps the most promising inflationary model, where the inflaton couples to the electromagnetic field in a dilaton-like manner. Dolgov [5] pointed out that this coupling was inevitably present due to the trace anomaly, although one would require a huge imbalance in the numbers of bosonic and fermionic degrees of freedom to generate the size of seed field that seems to be required.

More recently, Mazzitelli and Spedalieri [7] have shown that there are (gauge-invariant) models with higher-derivative couplings which produce large enough seed fields, although they involve derivative couplings of a rather high order. Still with inflation, Gasperini et al. [8] and Lemoine and Lemoine [6] used the fact that theories based on the low-energy limit of string theory are not conformally invariant in the electromagnetic sector because of the interaction with the dilaton, and thus could in principle generate large-scale fields during inflation, given a big enough change in the value of the dilaton field. However, the two sets of authors differed over whether a sufficiently large change occurs in the string-inspired

"pre-Big-Bang" scenario of Gasperini and Veneziano [9].

The first to suppose that a cosmological phase transition may generate a magnetic field appears to have been Hogan [10]. His idea was that an unspecified mechanism might operate during a first-order phase transition, which could generate a field with energy density of order the equipartition value, coherent on the scale determined by the size of bubbles of the low temperature phase. Hogan concentrated on the QCD phase transition at 0.2 GeV. Since then, several specific mechanisms for generating a seed field at first order phase transitions have been put forward [11–13]. The attractive possibility that the required seed field could be produced in the electroweak phase transition, even a continuous one, was first introduced by Vachaspati [14]. His estimate put the size of the field way below that necessary to supply the seed for galactic fields. However, subsequent estimates by another group [15] were several orders of magnitude larger, and they came to the opposite conclusion: that the electroweak phase transition *could* produce the required seed field. There is therefore some confusion about the subject.

It is the aim in this paper to try and remove some of this confusion concerning the generation of magnetic fields at the electroweak and other phase transitions. We begin by reprising Vachaspati's and Enqvist and Olesen's arguments. We discuss the definition of the magnetic field tensor in the electroweak theory, which is not unique, and the averaging procedures used by both sets of authors. We find that although Enqvist and Olsen perform the calculation of the line-averaged magnetic field correctly, the result is not the one required to estimate the size of the seed field. It is the *volume* average over galactic scales that is needed.

It is further pointed out that, independently of the definition of the electromagnetic field, one would in any case expect to find a thermal magnetic field at the transition, with correlation length $(eT)^{-1}$ (the inverse Debye mass). Any primordial seed field must be clearly distinguished both in scale and amplitude from the thermal fluctuations, which have evolved into the microwave background radiation, whose fields are negligible at galactic scales. It follows that any seed field must be generated by a non-equilibrium process. Energy conservation and causality put strong limits on the strength of fields generated at phase transitions independently of the details of the process.

II. MAGNETIC FIELDS FROM PHASE TRANSITIONS

In a phase transition in the early Universe, a multicomponent Higgs scalar field Φ (which may be fundamental or composite) acquires a vacuum expectation value of modulus $\eta \simeq T_C$, where T_C is the critical temperature at which the phase transition occurs. In the case of the electroweak transition, let W_μ^a ($a = 1, 2, 3$) be the three SU(2) vector boson fields and Y_μ the U(1) gauge boson, which have field strengths $F_{\mu\nu}^a$ and $F_{\mu\nu}^0$ respectively. The electromagnetic vector potential A_μ may then be defined by

$$A_\mu = -W_\mu^a \widehat{\phi}^a \sin\theta_W + Y_\mu \cos\theta_W, \qquad (1)$$

where θ_W is the Weinberg angle and $\widehat{\phi}^a = \Phi^\dagger \sigma^a \Phi / |\Phi|^2$ is a unit SU(2) vector, which is well-defined barring topological obstructions [16,17].

Using this unit isovector, we can also project out the linear combination of the field strength tensors associated with the electromagnetic field:

$$F_{\mu\nu}^{\text{em}} = -F_{\mu\nu}^a \widehat{\phi}^a \sin\theta_W + F_{\mu\nu}^0 \cos\theta_W.$$

It might seem natural that this is the field strength of electromagnetism: however, there is another gauge invariant quantity with the correct symmetry properties, namely

$$D_{\mu\nu} = \epsilon^{abc} \widehat{\phi}^a D_\mu \widehat{\phi}^b D_\nu \widehat{\phi}^c,$$

where $D_\mu \widehat{\phi}^a = (\partial_\mu + g\epsilon^{abc} W_\mu^b)\widehat{\phi}^c$. It is not immediately obvious how to combine the two quantities to make an electromagnetic field strength tensor. Vachaspati [14] follows 't Hooft's convention [18], which is to use

$$\mathcal{F}_{\mu\nu}^{\text{em}} = F_{\mu\nu}^{\text{em}} + \frac{\sin\theta_W}{g} D_{\mu\nu}. \qquad (2)$$

This has the advantage that it obeys the Bianchi identity (which includes the Maxwell equation $\nabla \cdot \mathbf{B} = 0$) almost everywhere, with the exception of isolated points where the isovector field vanishes with non-zero index. This, however, is not a particularly good criterion, as we know that magnetic charge exists in non-Abelian theories. A further argument against this definition come from the fact that the energy density of the electromagnetic field is no longer $\frac{1}{2}(\mathbf{E}^2 + \mathbf{B}^2)$. This serves to emphasise the point that the definition is more a matter of taste than physics, and arguments which rest on a particular choice are on shaky ground.

In any case, let us reproduce Vachaspati's argument for the production of magnetic fields by the electroweak phase transition. The minimum energy state of the Universe corresponds to a spatially homogeneous vacuum in which Φ is covariantly constant, i.e. $D_\mu \Phi = 0$, with $D_\mu = \partial_\mu - igW_\mu^a \tau^a - ig'Y_\mu$. It also follows that $D_\mu \widehat{\phi}^a = 0$. In this state there are no electromagnetic (or any other) excitations. However, immediately after the phase transition there is a finite correlation length ξ, which means that the Higgs field takes up random and independent directions in its internal space in regions of size $\sim \xi$. Thus, $D_\mu \widehat{\phi}^a \neq 0$. The correlation length is certainly less than the horizon length at T_C, and presumably much less. In reference [14] it is argued that these non-zero covariant derivatives give rise to a non-zero field $\mathcal{F}_{\mu\nu}^{\text{em}}$.

Suppose, therefore, we are interested in the magnetic field component averaged over a line segment $L = n\xi$. Vachaspati argues that, since the scalar field is uncorrelated on scales greater than ξ, its gradient executes a random walk as we move along the line, and so the average of $D_i\widehat{\phi}^a$ over n adjacent lattice points should scale as $1/\sqrt{n}$. He then concludes that, since the magnetic field is proportional to the product of two covariant derivatives of the Higgs field, it will scale as $1/n$. This latter conclusion, however, overlooks the difference between $(D_j\Phi^\dagger)\mathrm{rms}(D_k\Phi)\mathrm{rms}$ and $(D_j\Phi^\dagger D_k\Phi)\mathrm{rms}$. Enqvist and Olesen noticed this point, and produced a corrected estimate for the averaged field, $B \sim B_\xi/\sqrt{n}$, where B_ξ is the initial field strength on the scale ξ_i. The field strength is order m_W^2 at the electroweak phase transition, and it is correlated on the scale $\xi_\mathrm{i} \sim 1/m_W$. The scale corresponding to the initial coherence length at the current epoch is $\xi_\mathrm{i}/a_\mathrm{ew}$, with $a_\mathrm{ew} \sim 3 \cdot 10^{-15}$, while the field strength will have decreased in proportion to a_ew^2. Thus, with $L \sim 1$ Mpc, $n \sim 10^{25}$, and so the line-averaged field today is between 10^{-19} and 10^{-18} Gauss, seemingly of the right order of magnitude.

However, the significance of this result is open to doubt on two counts. Firstly, an objection has already been raised by Davidson [19], who pointed out that the physical Higgs field is neutral under $\mathrm{U}(1)_\mathrm{em}$, and therefore cannot directly generate electromagnetic currents. One should really regard $D_\mu\widehat{\phi}^a$ as the W field, to which it reduces in the unitary gauge. Nonetheless, electromagnetic fields can still be sourced by currents of W particles, for

$$\partial^\nu F_{\mu\nu}^\mathrm{em} = -\sin\theta_\mathrm{w} F_{\mu\nu}^a D^\nu\widehat{\phi}^a. \tag{3}$$

(There is an additional term on the right hand side for 't Hooft's definition). Secondly, as Enqvist himself points out [20], surface and volume averages of the field give much lower estimates. The physics cannot possibly depend on how we average the magnetic field when measuring it: thus we should examine what the different averages mean.

III. AVERAGING MAGNETIC FIELDS

For cosmological applications, we are interested in the value of the magnetic field on scales of order 1 Mpc, in some suitably averaged sense. It is not entirely obvious what one means by a magnetic field on a scale L: does one average over a line, a surface or a volume? Enqvist and Olesen [15] use the line average

$$B_{(1)} = \frac{1}{L}\int_C \mathbf{B}\cdot\mathbf{dx}, \tag{4}$$

where C is a curve, which we may take to be a straight line, of length L. (Vachaspati [14] also implicitly used this average, although in an incorrect way, as we have explained.) It is also possible to consider an average flux

$$B_{(2)} = \frac{1}{L^2}\int_S \mathbf{B}\cdot\mathbf{dS}. \tag{5}$$

Lastly, there is a volume averaged field, which is a vector,

$$\mathbf{B}_{(3)} = \frac{1}{L^3}\int_V \mathbf{B}\ d^3x. \tag{6}$$

We shall show that only the third of these has any information about the underlying magnetic field: the line and surface averages always deliver the same behaviour with scale L, regardless of how the field actually behaves.

We resolve the field into its Fourier components:

$$\mathbf{B}(\mathbf{x}) = \sum_\mathbf{k} \mathbf{B}_\mathbf{k} e^{i\mathbf{k}\cdot\mathbf{x}}.$$

In a very large region Ω, the summation goes over to the integral $\Omega\int d^3k/8\pi^3$. We then postulate that the field is statistically random and isotropic, with a correlation length ξ. We can therefore suppose that the average power spectrum goes as

$$P_\mathrm{B} \equiv \langle|\mathbf{B}_\mathbf{k}|^2\rangle \propto \frac{\xi^3}{\Omega} B_\xi^2 (k\xi)^{2p}, \qquad (k\xi \ll 1), \tag{7}$$

where $k = |\mathbf{k}|$. If $p \le -3/2$, there must be a lower cut-off on k, otherwise the energy density of the magnetic field diverges. The correlation length ξ provides the upper cut-off.

Firstly, we compute the r.m.s. line average (4) over the volume Ω. We take the curve C to be a line of length L in the x direction, centred at $\mathbf{r}$. Then

$$B_{(1)} = \sum_{k_1 k_2 k_3} W(k_1 L) B_1(k) e^{i\mathbf{k}\cdot\mathbf{r}}, \tag{8}$$

where W is a dimensionless window function. If we weight the entire length of the line equally, we have

$$W(k_1 L) = \frac{\sin(k_1 L/2)}{k_1 L/2}. \tag{9}$$

Assuming that statistical averages are identical to space averages, we find

$$\left\langle B_{(1)}^2\right\rangle = \sum_{k_1 k_2 k_3} W^2(k_1 L)|B_1(k)|^2. \tag{10}$$

Defining $\kappa^2 = k_2^2 + k_3^2$, we have

$$\left\langle B_{(1)}^2\right\rangle \sim \frac{B_\xi^2}{\xi^3}\int dk_1 W^2(k_1 L)\int\langle P_\mathrm{B}(\kappa^2 + k_1^2). \tag{11}$$

Provided that $p > -3/2$, this integral is well-defined, and for $L \gg \xi$, we obtain

$$\left\langle B_{(1)}^2\right\rangle \sim B_\xi^2\frac{\xi}{L}. \tag{12}$$

Thus *any* power spectrum with $p > -3/2$ will give the same result, that the r.m.s. line-averaged magnetic field decreases as $L^{-1/2}$ as the scale L increases.

In any case, the line average is not a very useful tool for describing an averaged magnetic field, as it reveals nothing about the power spectrum of the magnetic field. The same is true of the surface average, for similar manipulations give for the average flux

$$\left\langle B_{(2)}^2 \right\rangle \sim B_\xi^2 \frac{\xi^2}{L^2}, \tag{13}$$

again with the proviso that $p > -3/2$. It is only the volume-averaged field contains any useful information in general, for in this case

$$\left\langle B_{(3)}^2 \right\rangle \sim k^3 \langle |\mathbf{B_k}|^2 \rangle \big|_{k=2\pi/L} \sim B_\xi^2 \left(\frac{\xi}{L} \right)^{(2p+3)}. \tag{14}$$

Hogan [10] made a similar point: to calculate the average flux on a scale L, one must smear the surface of area L^2 on a scale L, otherwise the average merely picks out the irrelevant high frequency power. The dynamics of the magnetic field are expressed in terms of differential equations for the individual Fourier components $\mathbf{B_k}(t)$: the "seed fields" are then clearly initial conditions for these Fourier components on the appropriate scale.

The line average *is* important when calculating the rotation measure of distant radio sources, which depends on $\int n_e \mathbf{B} \cdot d\mathbf{x}$ along the line of sight, where n_e is the free electron density [1]. From this can be derived upper bounds on cosmological magnetic fields [23].

What values of p can we expect? In a thermal electroweak phase transition, as envisaged by Vachaspati, there are several arguments which all give $p = 0$. His original idea was that the mixing between the SU(2) gauge fields and the hypercharge field allowed electric currents, and hence magnetic fields, to be generated by gradients of the Higgs field. The Higgs field is correlated on some scale ξ_H, but not above, and so therefore is the magnetic field. Averaging $\mathbf{B}$ over a volume L^3, with $L \gg \xi$, one finds that the individual correlated regions of field add randomly, and therefore $\int_{L^3} d^3x \mathbf{B}$ grows only as $(L/\xi_H)^{3/2}$. Hence $\left\langle B_{(3)}^2 \right\rangle \sim L^{-3}$, which corresponds to $p = 0$. One can also suppose, as did Enqvist and Olesen, that the magnetic field has its own correlation length, and therefore the total field averaged over a set of correlation volumes adds as a random walk. This results in the same answer, $p = 0$. Lastly, $p = 0$ is precisely what one obtains from an electromagnetic field in thermal equilibrium. To see this, let us consider the energy density in photons up to a frequency $\omega \sim L^{-1}$, which is

$$\rho_\gamma(L) \sim \int_0^{L^{-1}} d\omega \omega^3 \frac{1}{e^{\omega/T} - 1}. \tag{15}$$

For $LT \gg 1$, we find

$$\rho_\gamma(L) \sim TL^{-3}. \tag{16}$$

However, as the energy is equally distributed between the electric and magnetic fields, we have also

$$\rho_\gamma(L) \sim \sum_{\mathbf{k}}^{L^{-1}} |\mathbf{B_k}|^2. \tag{17}$$

Thus a volume-averaged magnetic field in thermal equilibrium has a power spectrum (7) with $p = 0$.

Lastly in this section, we note that a scale invariant spectrum, as might be generated in an almost-de Sitter inflationary model [4], corresponds to $p = -3/2$, for which we must also specify a long-wavelength cut-off Λ. In this case, all the averages have the same behaviour with L,

$$\langle B_{(a)}^2 \rangle \sim B_\xi^2 \ln(\Lambda/L). \tag{18}$$

IV. CAUSAL GENERATION OF MAGNETIC FIELDS IN THE RADIATION ERA

The fact that fields in equilibrium have the same power spectrum as that obtained by Vachaspati's mechanism leads one to ask how one might distinguish a seed field field generated at the phase transition from the background thermal fluctuations. It is natural to suppose that at the phase transition a linear combination of the fluctuations in the W and Y fields will emerge as thermal fluctuations in the electromagnetic field, with a scale $(eT)^{-1}$. We should therefore be careful to distinguish them from any putative seed field. A seed field should have a scale much greater than the thermal fluctuation scale, otherwise it would be absurd to talk of it being "frozen in" [21]. Its amplitude in each Fourier mode should also be greater. A transition which remains in thermal equilibrium cannot generate anything other than thermal fluctuations, which are cosmologically interesting only in that they evolve into the cosmic microwave background. We must therefore invoke a departure from thermal equilibrium by, for example, a first order phase transition. This can certainly generate a scale quite different than $(eT)^{-1}$ via the average bubble size when the low temperature phase percolates. It is not clear, however, how much of the energy of the bubbles can be transferred into the magnetic field.

Henceforth we free ourselves from any particular mechanism and try to see if there are bounds on primordial fields from processes operating in the radiation era from general principles such as energy conservation. Let us suppose that some process at time t_i creates a magnetic field of strength B_i on a scale ξ_i. Causality demands that $\xi_i < t_i$ and that the power spectrum goes as k^n, with $n \geq 0$, at small k, and energy conservation that $B_i^2 < \rho(t_i)$, the total energy density. If the field were completely frozen in, the power spectrum would remain of the form (7), with the coherence scale of the field ξ fixed in comoving coordinates. The energy density (or

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equivalently the mean square fluctuation) on a scale L greater than $\xi = a(t)\xi_i/a_i$ is given by

$$B_{\mathrm{fr}}^2(t, L) \simeq B_i^2 \left(\frac{a_i}{a(t)} \right)^4 \left(\frac{a(t)\xi_i}{a_i L} \right)^{3+n}. \qquad (19)$$

This is of course an idealisation: nonetheless $B_{\mathrm{fr}}(t, L)$ is a useful reference value.

Consider the ratio

$$r(L) = B^2(t_0, L)/8\pi\rho_\gamma(t_0), \qquad (20)$$

where ρ_γ is the photon energy density. From the requirement that the magnetic energy density not exceed the total,

$$r_{\mathrm{fr}} < \frac{1}{g_*(t_i)} \left(\frac{\xi_i}{a_i L} \right)^{3+n} \qquad (21)$$

where $g_*(t)$ is the effective number of relativistic degrees of freedom at time t. Thus

$$r_{\mathrm{fr}}(L) < \frac{1}{g_*(t_i)} \left(\frac{T_{\mathrm{eq}}}{T_i} \right)^{3+n} \left(\frac{\lambda_{\mathrm{eq}}}{L} \right)^{3+n}, \qquad (22)$$

where λ_{eq} is the comoving horizon scale at t_{eq}, the time of equal matter and radiation densities. With $H_0 = 50\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, $T_{\mathrm{eq}} \simeq 1\,\mathrm{eV}$ and $\lambda_{\mathrm{eq}} \simeq 50\,\mathrm{Mpc}$.

For the QCD phase transition, $T_i \sim 10^8\,\mathrm{eV}$, and for the electroweak transition $T_i \sim 10^{11}\,\mathrm{eV}$. Thus, for causal processes happening at these transitions we obtain the upper bounds on frozen-in fields

$$r_{\mathrm{fr}}(0.5\,\mathrm{Mpc}) < \begin{array}{ll} 10^{-19-6n} & \text{(QCD transition)}, \\ 10^{-29-9n} & \text{(Electroweak transition)}, \end{array}$$
$$(23)$$

where we have taken g_* to be 10 and 100 respectively. Given that $r = 1$ corresponds to a field strength of $3 \cdot 10^{-6}$ Gauss, the electroweak transition can at best manage about 10^{-20} Gauss on the 0.5 Mpc scale. Thus it can be seen that it is energetically possible that some causal process operating at the time of the electroweak phase transition could produce a frozen-in seed field. However, nearly the energy density of the Universe would have to be converted into a field coherent at the horizon scale, which seems very hard to arrange. It is perhaps more realistic to suppose that the equipartition energy can be transferred to a field coherent on 1/10 the horizon scale, from which we obtain an upper bound of 10^{-25} Gauss.

Magnetic fields are of course not completely frozen into the plasma. The field can drag matter around with it, and field lines can pass through each other by reconnection. The result is a power-law increase in the comoving scale of the field [10]. However, any process which merely increases the scale ξ will actually weaken the field on larger scales: in order to generate stronger fields one must transfer power from small to large scales. By causality, this process cannot operate on scales greater than the horizon size $\sim t$, so the power transfer, if it happens, must take power from below the coherence scale ξ and inject into scales between ξ and t.

A set of model MHD equations was shown to have just such an "inverse cascade" by Brandenburg et al. [22]. However, in the absence of solutions to the full MHD equations one cannot be fully confident that a primordial field would also show such behaviour.

V. CONCLUSIONS

This paper has focused on the generation of fields at the electroweak phase transition, but we have arrived at some more general results. Firstly, we have tackled the confusion surrounding the concept of the average magnetic field on a scale L, showing that the quantity of dynamical interest is the *volume* average of the field. Secondly, we have re-examined Vachaspati's proposal for generating fields at the electroweak phase transition. It was shown that the argument is based on a particular definition of the electromagnetic field in the Standard Model, which is not unique. In any case, one must be careful to distinguish any random magnetic seed field from a thermal field. The remnant of the thermal radiation at the electroweak transition is around us in the form of the Cosmic Microwave Background: Thus a continuous electroweak phase transition, where there is essentially no departure from thermal equilibrium, produces negligible fields on galactic scales. Thirdly, we saw that on grounds of causality and energy conservation alone, the idea of generating a seed field at or before the electroweak transition is quite unlikely, unless there is some extra physics in the form of an inverse cascade in the turbulent magnetic field, which transfers energy from small-scale fluctuations in the field to large ones. The QCD transition is not so strongly constrained, and mechanisms exist based on charge separation in advanced phase boundaries [12].

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